Engineering Notes

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Active Stabilization of a Dual-Spin Spacecraft using Imbalance

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Nomenclature

X, Y, Z = body fixed orthogonal set of axes
X_p, Y_p, Z_p = body fixed principal axes
I_1, I_2, I_3 = moments of inertia values of the spacecraft along X, Y and Z respectively; Z is the common spin-axis of the spacecraft.
I_r = moment of inertia of the rotor about the common spin axis.
$\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3$ = body fixed velocity vectors of the despun body.
Ω = rotor velocity about the body fixed common spin axis.
τ = modulating torque in the pitch axis control system.

Introduction

Much literature is available on the stability of dual-spin spacecraft using various damper configurations.[1,2] These necessitate the development of an extra component, i.e., the nutation damper. Philips[3] introduced a unique technique by placing an imbalance on the despun part and introducing a modulating signal on the despin control system. A pick-up was used to sense the transverse velocity. The modulating signal had a phase difference with respect to the pick-up signal. The best value of the phase difference was computed by estimating the damping decay time at various phase angles. The technique is of enormous practical utility but the treatment does not deal with dynamic aspects such as how imbalance helps in stabilizing the system, estimation of time constants, etc.

This Note considers a dual-spin spacecraft with imbalance on the despun part in the $Y-Z$ plane. The spacecraft has a pick-up at an angle β in the $X-Y$ plane and a modulating signal proportional to the pick-up in the despin motor, providing modulating torque in the despin control system. A linearized model is analyzed first and the energy sink analysis is considered later. Results of both analysis are compared with the numerical integration of the exact equations.

The following assumptions are made in the analysis: 1) Imbalance on the despun part is small, so the basic nature of the spacecraft motion is not considerably affected. It is also assumed that the principal axis Z_p lies in the $Y-Z$ plane, such that, imbalance can be defined by a single parameter, I_{23}, i.e., $I_{12}=I_{13}=0$; 2) Modulating torque, τ is small, such that, the pitch axis error caused is within limits and, also, the change in rotor speed is negligible; and 3) The parameters are selected in such a way that the spacecraft is otherwise dynamically stable.

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Linear Analysis

The basic equations of motion, derived from Euler's equations, for a dual-spin spacecraft are

$$I_1\dot{\mathbf{W}}_1-(I_2-I_3)\mathbf{W}_2\mathbf{W}_3+I_{23}(\mathbf{W}_3^2-\mathbf{W}_2^2)+I_r\Omega\mathbf{W}_2=0 \quad (1)$$

$$I_2\dot{\mathbf{W}}_2-(I_3-I_1)\mathbf{W}_3\mathbf{W}_1-I_{23}\dot{\mathbf{W}}_3+I_{23}\mathbf{W}_1\mathbf{W}_2-I_r\Omega\mathbf{W}_1=0 \quad (2)$$

$$I_3\dot{\mathbf{W}}_3-(I_1-I_2)\mathbf{W}_1\mathbf{W}_2+I_{23}\mathbf{W}_1\mathbf{W}_3-I_{23}\dot{\mathbf{W}}_2+I_r\dot{\Omega}=0 \quad (3)$$

$$I_r\dot{\Omega}=\tau \quad (4)$$

While considering the stability, the pitch axis control torques will not be considered. Let us consider the modulating torque as a function of two transverse velocities

$$\tau=T(A_1\mathbf{W}_1+A_2\mathbf{W}_2) \quad (5)$$

Where T is a constant. Moreover A_1 and A_2 are constants such that $A_1^2+A_2^2=1$. The aim of the following analysisis to evaluate the best proportion of A_1 and A_2 so as to damp the nutations at the fastest rate.

Simplifying Eqs. (1-3) with the assumption that the quantities $\mathbf{W}_1$, $\mathbf{W}_2$ and I_{23} are very small,

$$\dot{\mathbf{W}}_1-\mathbf{W}_2\left(\mathbf{W}_3\frac{I_2-I_3}{I_1}-\frac{\Omega I_r}{I_1}\right)+\frac{I_{23}}{I_1}\mathbf{W}_3^2=0 \quad (6)$$

$$\dot{\mathbf{W}}_2-\mathbf{W}_1\left(\mathbf{W}_3\frac{I_3-I_1}{I_2}+\frac{\Omega I_r}{I_2}\right)+\frac{I_{23}T}{I_2I_3}(A_1\mathbf{W}_1+A_2\mathbf{W}_2)=0 \quad (7)$$

$$\dot{\mathbf{W}}_3+\frac{T}{I_3}(A_1\mathbf{W}_1+A_2\mathbf{W}_2)=0 \quad (8)$$

Since T is very small and the nature of the modulating torque is oscillatory $\mathbf{W}_3$ can be considered as constant for the purpose of linearizing the Eqs. (6) and (7). Substituting

$$-\left(\mathbf{W}_3\frac{I_2-I_3}{I_1}-\frac{I_r\Omega}{I_1}\right)=K_1, \quad (9a)$$

$$\left(\mathbf{W}_3\frac{I_3-I_1}{I_2}+\frac{I_r\Omega}{I_2}\right)=K_2 \quad (9b)$$

into Eqs. (6) and (7) and taking the Laplace transform, the characteristic equation associated with these equations is

$$S^2+S\,A_2T\frac{I_{23}}{I_2I_3}+K_1K_2-K_1A_1T\frac{I_{23}}{I_2I_3}=0 \quad (10)$$

Roots of the characteristics equations are

$$S=-\frac{TA_2I_{23}}{2I_2I_3}\pm\tfrac{1}{2}\left[\left(\frac{TA_2I_{23}}{I_2I_3}\right)^2-4K_1K_2+\frac{4K_1TA_1I_{23}}{I_2I_3}\right]^{1/2} \quad (11)$$

For an otherwise stable system as in assumption (3) $K_1K_2>0$. The presence of imbalance has the following effects. 1) The frequency of nutations is modified from $\Omega_N=(K_1K_2)^{1/2}$ to

$$\Omega_N=\left[K_1K_2-\left(\frac{TA_2I_{23}}{2I_2I_3}\right)^2-\frac{K_1TA_1I_{23}}{I_2I_3}\right]^{1/2}$$

Since I_{23} and T are small quantities, the variation in nutational frequency is insignificant. 2) The magnitude of nutation damps out with the time constant

$$\tau_{c1}=2I_2I_3/TA_2I_{23} \tag{12}$$

Where the damping time is independent of A_1 and directly depends on A_2, hence the modulating signal should be in phase with $\mathbf{W}_2$ ($A_2=1.0$, $A_1=0$). As a result, the modulating signal should be in phase with the transverse velocity about the axis in the plane of imbalance. This implies $\beta=0$ (Fig. 1). The damping is directly proportional to the amount of imbalance and finally, the damping is directly proportional to the magnitude of modulating torque.

Energy Sink Analysis

In the preceeding analysis, it was shown through linearized equations that periodic motion is achieved about the body fixed axis. The magnitude variations of $\mathbf{W}_1$ and $\mathbf{W}_2$ are very slow and will remain practically constant over the duration of one cycle. The basic nature of the motion can be obtained by substituting $\tau=I_{23}=0$. From Eqs (6-9)

$$\dot{\mathbf{W}}_1+K_1\mathbf{W}_2=0,\quad \dot{\mathbf{W}}_2-K_2\mathbf{W}_1=0 \tag{13}$$

With the initial conditions,

$$\mathbf{W}_1(0)=0\quad \mathbf{W}_2(0)=\mathbf{W}_T$$

the solution of Eq. (13) is

$$\mathbf{W}_1=-\mathbf{W}_T\left(\frac{K_1}{K_2}\right)^{1/2}\sin(K_1K_2)^{1/2}t,\quad \mathbf{W}_2=\mathbf{W}_T\cos(K_1K_2)^{1/2}t \tag{14}$$

The square of the angular momentum of the system about the mass center is

$$H^2=\bar{h}.\bar{h}=(I_1\mathbf{W}_1)^2+(I_2\mathbf{W}_2-I_{23}\mathbf{W}_3)^2$$
$$+(I_3\mathbf{W}_3+I_r\Omega-I_{23}\mathbf{W}_2)^2 \tag{15}$$

As there are no external torques, the total angular momentum remains constant, so that

$$\frac{d}{dt}H^2=2I_1^2\mathbf{W}_1\dot{\mathbf{W}}_1+2(I_2^2+I_{23}^2)\mathbf{W}_2\dot{\mathbf{W}}_2$$
$$-I_2I_{23}(\dot{\mathbf{W}}_3\mathbf{W}_2+\dot{\mathbf{W}}_2\mathbf{W}_3)+2I_{23}^2\mathbf{W}_3\dot{\mathbf{W}}_3$$
$$+2(I_3\mathbf{W}_3+I_r\Omega)(I_3\dot{\mathbf{W}}_3+I_r\dot{\Omega})$$
$$+2I_{23}\dot{\mathbf{W}}_2(I_3\mathbf{W}_3+I_r\Omega)$$
$$-2I_{23}\mathbf{W}_2(I_3\dot{\mathbf{W}}_3+I_r\dot{\Omega})=0 \tag{16}$$

The magnitude variation of $\mathbf{W}_T$ is very slow compared to one cycle period. Averaging Eq. (16) over a period $2\pi/(K_1K_2)^{1/2}$ and assuming that there is no dissipation in the rotor despun part, all but secular terms will be filtered out, so that

$$\dot{\mathbf{W}}_T=-\frac{I_{23}TA_2}{(K_1/K_2)I_1^2+I_{23}^2+I_2^2}\mathbf{W}_T \tag{17}$$

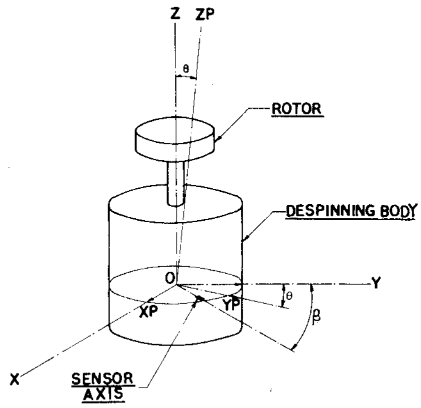

Fig. 1 Dual-spin system.

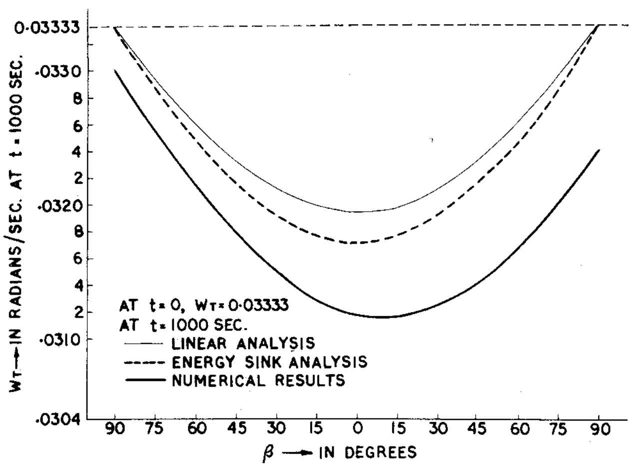

Fig. 2 Values of $\mathbf{W}_T$ at $t=1000$ sec.

Equation (17) yields the damping time constant

$$\tau_{c2}=\frac{I_1^2(K_1/K_2)+I_{23}^2+I_2^2}{I_{23}TA_2} \tag{18}$$

Equations (12) and (18) are different suggesting that the results obtained from the preceeding approximate methods are not the same. However, the denominator is the same and we can conclude that the three remarks made earlier remain valid. Considering an axisymmetric spacecraft, $I_1=I_2$ the evaluated time constants are in proportion.

Numerical Results

The dynamic behavior of the exact system was studied by numerically integrating the set of Eqs. (1-4) using a fourth-order Runge-Kutta method. The equations were integrated for 1000 seconds real time with the following typical data.

$$I_1=420\ kg.M^2\quad I_2=420\ kg.M^2\quad I_3=360\ kg.M^2$$

$$I_r=5.0\ kg.M^2\quad I_{23}=5.0\ kg.M^2\quad T=3.0$$

$$\mathbf{W}_{T0}=0.03333\ \text{rad./sec.}\quad \Omega=3.14\ \text{rad./sec.}$$

A_1 and A_2 are so adjusted that the modulating signal maintains a phase difference with the vector $\mathbf{W}_2$. The phase difference is varied from $-90°$ to $+90°$ and the value of the vector $\mathbf{W}_T$ is computed at $t=1000$ secs. The graph (Fig. 2) gives the values of $\mathbf{W}_T$ at $t=1000$ secs, for (1) linear analysis, (2) energy sink method, and (3) numerical results. The results are quite close and the small variations arise as a result of the simplifying assumptions made during the analytical study.

The best phase relationship obtained from the numerical results is near $\beta = 0$, however, this is close enough to the generalized results and the time constant will not vary much near the optimum value, being proportional to $1/\cos\beta$, β near zero.

Stability in Presence of Dissipation in the Rotor

In this Section, the energy sink analysis for dual-spin spacecraft used by Likins[4] is extended to evaluate the stability conditions in the presence of dissipation in the rotor. Such a configuration is unstable if there is no dissipation in the despun body. Presence of modulating torque and imbalance can be used to stabilize such a system. The total rotational kinetic energy is

$$T = \tfrac{1}{2}(I_1 \mathbf{W}_1^2 + I_2 \mathbf{W}_2^2 + I_3 \mathbf{W}_3^2 - 2I_{23}\mathbf{W}_2\mathbf{W}_3 + I_r\Omega^2)$$

so that the rate of energy dissipation is

$$\dot{T} = I_1\mathbf{W}_1\dot{\mathbf{W}}_1 + I_2\mathbf{W}_2\dot{\mathbf{W}}_2 + I_3\mathbf{W}_3\dot{\mathbf{W}}_3$$
$$+ I_r\Omega\dot{\Omega} - I_{23}(\mathbf{W}_3\dot{\mathbf{W}}_2 + \mathbf{W}_2\dot{\mathbf{W}}_3)$$

substituting $\mathbf{W}_1$, $\mathbf{W}_2$ from Eq. (14) and proceeding as in Ref. 4 one obtains

$$\mathbf{W}_T\dot{\mathbf{W}}_T = \left(\frac{P_A}{\lambda_A} + \frac{P_s}{\lambda_s}\right)\left(\frac{2h_0 K_2}{I_1^2 K_1 + I_2^2 K_2}\right) - \frac{I_{23}\, T A_2\, \mathbf{W}_T^2}{(K_1/K_2)I_1^2 + I_2^2 + I_{23}^2}$$

where

$$\lambda_0 = h_0(I_1K_1 + I_2K_2)/(I_1^2K_1 + I_2^2K_2)$$

$$\lambda_A = \lambda_0 - \mathbf{W}_3, \quad \lambda_s = \lambda_0 - \Omega$$

P_A = average dissipation rate in the despun body

P_s = average dissipation rate in the rotor

For the spacecraft to be stable, the right hand side must be less than zero. In case $P_A = 0$, we have for stability

$$\left|\frac{P_s}{\lambda_s}\left[\frac{2h_0 K_2}{I_1^2K_1 + I_2^2K_2}\right]\right| < \left|\frac{I_{23}TA_2\mathbf{W}_T^2}{(K_1/K_2)I_1^2 + I_2^2 + I_{23}^2}\right|$$

In most cases, the average dissipation in rotor will be proportional to $\mathbf{W}_T^2$, say, $P_S = K.\ \mathbf{W}_T^2$. Then we have for stability

$$\left|\frac{K}{\lambda_s}\ \frac{2h_0 K_2}{I_1^2K_1 + I_2^2K_2}\right| < \left|\frac{I_{23}\,TA_2}{(K_1/K_2)I_1^2 + I_{23}^2 + I_2^2}\right|$$

where dissipation rate in the rotor is constant, or if it is some other function of $\mathbf{W}_T$, sustained oscillations may result with the $\mathbf{W}_T$ such that

$$\left|\frac{P_s^*}{\lambda_s}\left[\frac{2h_0 K_2}{I_1^2K_1 + I_2^2K_2}\right]\right| = \left|\frac{I_{23}TA_2\mathbf{W}_{Ts}^2}{(K_1/K_2)I_1^2 + I_2^2 + I_{23}^2}\right|$$

where Ps^* is the energy dissipation rate in the rotor at $\mathbf{W}_T = \mathbf{W}_{TS}$

Conclusion

The presence of a modulating signal in the pitch axis control system of a dual-spin stabilized spacecraft with imbalanced despun parts can be used for effective stabilization by making the modulating torque in phase with the body fixed transverse velocity aligning with the axis of imbalance.

References

[1] Vigneron, F. R., "Stability of a Dual-Spin Satellite with Two Dampers," *Journal of Spacecraft and Rockets*, April 1971, pp. 386-389.

[2] Mingori, D. L., Harrison, J. A., and Tseng, C. T., "Semi-Passive and Active Nutation Damper for Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets*, May 1971, pp. 448-455.

[3] Philips, K. J., "Active Nutation Damping Utilizing Spacecraft Mass Properties," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-9, Sept. 1973, pp. 688-693.

[4] Likins, P. W., "Attitude Stability Criteria for Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 4, Dec. 1967, pp. 1638-1643.

Geometric Theory of Horizon Scanners

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Nomenclature

[ ]	= matrix
$[\ ]^T$	= matrix transpose
$[A_{I/B}]$	= $[a_{ij} = \cos(X_i^I, X_j^B)]$, direction cosine matrix relating a set of spacecraft axes X_j^B to a set of inertial axes X_i^I, $i, j = 1, 2, 3$. Similar notation is used to relate other coordinate axes.
$[(\ )^N]$	= a vector ( ) expressed in terms of its components in the X_i^N axes.

Introduction

HORIZON scanners constitute a most important class of spacecraft attitude sensors. A brief description of the different kinds of horizon scanners, together with references, may be found in Hatcher.[1] Most of the studies on horizon sensors were devoted to the hardware, the electronics and the propagation aspects. Although there exists scattered proprietary information, there is a lack of a systematic and critical study of the geometry of horizon measurements from which spacecraft attitudes are to be determined. This is perhaps not of much significance when horizon sensor outputs are used for attitude stabilization purposes. For the purpose of attitude determination the situation is different. When the Earth is in view, the information content of horizon scanner outputs is large because virtually a useful measurement is made for each revolution of the scanner. All these measurements are meaningful for precise attitude determination based on statistical data processing procedures. It is the purpose of this Note to present a general geometrical theory of horizon scanners.

Analysis

A scanner generates a scanning ray or vector. The scanning action may be derived from the spin of a spin-stabilized spacecraft for a body-mounted instrument. More often the scanner rotates either mechanically or optically about an axle which has a fixed orientation in the spacecraft. In panoramic scanners, the direction of the scanner axle may be maneuvered on command. In any case, the direction of the scanning vector relative to the spacecraft is some known function of time. A

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